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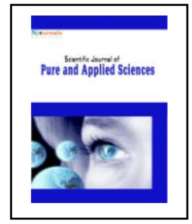

Original article

A novel algorithm for solving two-objective fuzzy transportation problems

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A new method is proposed for finding efficient solution sets for fuzzy two objective transportation problems using ranking function and percent of function one solution is introduced for transportation problem and explained with the proposed model. Decision maker can obtain efficient solutions with the proposed method and selects the most preferred one among them.

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1. Introduction

Objective of classic transportation problem is minimizing cost but generally, problems which are formulated are multi-purpose and these purposes will be measured in different scales and are inconsistent with each other. In practice, ideal solution for multi-objective problem is always impossible. When objectives are in conflict with each other, we cannot reach these solutions. For this purpose, efficient solution will be introduced instead of ideal solution. Often, objective functions and right-hand columns are considered as fuzzy data in multi-objective transportation problems that these values are determined by decision-maker; then, he will reach solution by analyzing data with required method. In 1979, an algorithm was introduced by Izerman for multi-objective transportation problems. in 2005, an algorithm was proposed by Omar and Yunes for solving multi-objective transportation problems using fuzzy factors. In 2011, a new method was presented by Pendian for solving two-objective transportation problem. In this article, a new method is suggested for finding efficient solution sets for two-objective fuzzy transportation using ranking function. In proposed method, solution of next problem could be found with current solution which is different from previous methods and satisfaction level is introduced for two-objective fuzzy problem. Finally, the method will be explained by numerical method.

2. Definitions

Definition 1.2 (fuzzy number): let R be real numbers set. $\tilde{a}$ fuzzy number is a map $\mu_{\tilde{a}}: R \to [0,1]$ with below conditions:

$\mu_{\tilde{a}}$ is continuous.

$\mu_{\tilde{a}}$ on $[a_1, a_2]$ is ascending and continuous.

$\mu_{\tilde{a}}$ on $[a_3, a_4]$ is descending and continuous.

That a_1, a_2, a_3 and a_4 are real numbers and fuzzy numbers is shown as $\tilde{a} = [a_1, a_2, a_3, a_4]$ and it is called trapezoid fuzzy number.

Definition 2.2 (membership function): if $\tilde{a}$ is trapezoid fuzzy number, membership function is as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & a_1 \le x \le a_2 \\ 1 & a_2 \le x \le a_3 \\ \dfrac{x - a_4}{a_3 - a_4} & a_3 \le x \le a_4 \end{cases}$$

Definition 2.3 (α-level set): α-level set of a fuzzy set $\tilde{a}$ is shown by a_α and are defined as below:

$$a_\alpha = \{x \mid \mu_{\tilde{a}}(x) \ge \alpha\} = [a_\alpha^l, a_\alpha^u] \qquad (1)$$

Definition 2.4 (ranking function): a suitable method for comparing fuzzy numbers is using ranking function. Ranking function $R: F(R) \to R$ is image of each fuzzy number on real numbers that $F(R)$ is set of all R fuzzy numbers and is as follows:

$$R(\tilde{a}) = \frac{1}{2}\int_0^1 (a_\alpha^l + a_\alpha^u)\, d\alpha \qquad (2)$$

Definition 5.2: let $\tilde{a}$ and $\tilde{b}$ are two fuzzy in F®, then

1) $\tilde{a} \ge \tilde{b}$ if and only if $R(\tilde{a}) \ge R(\tilde{b})$;
2) $\tilde{a} > \tilde{b}$ if and only if $R(\tilde{a}) > R(\tilde{b})$;
3) $\tilde{a} = \tilde{b}$ if and only if $R(\tilde{a}) = R(\tilde{b})$;

3. Two-objective transportation problems

Two-objective transportation problem has a mathematical model:

$$(P)$$

$$\text{Min} \quad Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}$$

$$\text{Min} \quad Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}\, x_{ij}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} = a_i \quad i = 1, \dots, m \qquad (3)$$

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad j = 1,2,\dots,n \qquad (4)$$
$$x_{ij} \geq 0 \quad i = 1,\dots,m \; ; \; j = 1,\dots,n \quad (5)$$
$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \qquad (6)$$

where a_i in product amount in ith source and b_j is amount of demand in jth destination and c_{ij} is cost of transferring one unit product from source i to destination j. d_{ij} cost of loss in product unit route from source I to destination j. x_{ij} is amount of transferring product from source i to destination j.

Definition 3.1: $X^0 = \{x_{ij}^0 : i = 1,\dots m; j = 1,\dots,n\}$ set has solution for P problem if it holds in conditions (3), (4) and (5).

Definition 3.2: X^0 is an efficient solution for problem (P) if there was no X solution for BTP such that $Z_1(X) \leq Z_1(X^0)$ and $Z_2(X) < Z_2(X^0)$ and $Z_1(X) < Z_1(X^0)$; otherwise, it is an inefficient solution for problem (P).

Definition 3.3: objective function satisfaction level is U solution for transportation system is defined as below:

$$\left(1 - \frac{O(U) - O_0}{O_0}\right) \times 100 = \left(\frac{(2O_0 - O(U))}{O_0}\right) \times 100 \quad (7)$$

where O(U) is objective function in solution U and O_0 is optimized solution of transportation problem.

Definition 3.4: (preferred solution): is efficient solution which is selected by decision maker as final decision.

4. two-objective fuzzy transportation problem

Mathematical model for fuzzy two-objective transportation problem is as follows when objective functions are fuzzy functions and demand and supply are real numbers:

$$\text{(FP)}$$
$$\text{Min} \quad \tilde{Z}_1 = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} x_{ij}$$
$$\text{Min} \quad \tilde{Z}_2 = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{d}_{ij} x_{ij}$$
$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} = a_i \quad i = 1,2,\dots,m \qquad (8)$$
$$\sum_{i=1}^{m} x_{ij} = b_j \qquad j = 1,2,\dots,n \qquad (9)$$
$$x_{ij} \geq 0 \quad i = 1,\dots,m; \; j = 1,\dots,n \quad (10)$$
$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \qquad (11)$$

5. Proposed method

Proposed algorithm is as follows:

Step 1: convert fuzzy two-objective transportation problem into two single-objective problems and name them in fuzzy transportation problem with FFOTP and SFOTP.

Step 2): obtain optimized solutions for FFOTP and SFOTP using fuzzy transportation algorithm (in order to solve fuzzy single objective problem, obtain fuzzy solution with Vogel-fuzzy method and then optimize it with fuzzy modified distribution method).

Step 3): FFOTP optimized solution is a possible solution for SFOTP and an efficient solution for FP. Then begin with FFOTP in SFOTP.

Step 4): select full cell (t,r) with maximum cost in SFOTP. Then form rectangular ring such that beginning and end corner is (t,r) cell and one corner is empty and others are full.

Step 5): add λ to a corner in empty cell and alternatively add λ to other corners. Then create a sequence of λ values such that non-negative full cells remain. One possible solution is obtained for each SFOTP which is an efficient or inefficient solution for FP.

Step 6) SFOTP is possible solution of step (5), go to next step, otherwise repeat steps (4) and (5) to obtain SFOTP.

Step 7): begin with SFOTP in FFOTP (optimal SFOTP solution, is possible solution for FFOTP and efficient solution for FP).

Step 8) repeat steps (4) and (5) for FFOTP.

Step 9) obtained solutions from FFOTP and SFOTP and efficient and inefficient solutions for two-objective transportation problem.

6. Descriptive example

Table 1

Consider a problem with below characteristics:(FFOTP).

destination → **Source↓**	**1**	**2**	**3**	**4**	**supply**
1	(0,1,1,2)	(0,1,3,4)	(4,6,8,10)	(3,6,9,10)	8
2	(0,1,1,2)	(6,7,10,13)	(1,2,4,5)	(2,3,5,6)	19
3	(4,6,9,13)	(4,8,10,14)	(1,4,5,6)	(2,5,6,11)	17
demand	11	3	14	16	

(SFOTP)

destination → **source↓**	**1**	**2**	**3**	**4**	**supply**
1	(2,3,5,6)	(1,4,5,6)	(1,2,4,5)	(1,3,5,7)	8
2	(2,4,5,9)	(6,7,9,10)	(6,8,10,12)	(8,9,11,12)	19
3	(4,5,7,8)	(0,1,3,4)	(2,4,6,8)	(0,1,1,2)	17
demand	11	3	14	16	

Table (2)

FFOTP solution is:

$$x_{11} = 5 \ ,x_{12} = 3\ ,x_{21} = 6\ ,x_{24} = 13\ ,$$
$$x_{33} = 14\ ,x_{34} = 3$$

Minimum transportation cost is:

$$\tilde{Z}_1^* = (46,124,173,229) \quad و \quad R(\tilde{Z}_1^*) = 143$$

SFOTP solution is:

$$x_{13} = 8 \ ,x_{21} = 11\ ,x_{22} = 2, x_{23} = 6\ ,$$
$$x_{32} = 1\ ,x_{34} = 16$$

And minimum transportation cost is:

$$\tilde{Z}_1^* = (89,147,176,256) \quad و \quad R(\tilde{Z}_1^*) = 167$$

Table 3

Now, regarding step (3), FFOTP solution is feasible solution for SFOTP in table 3.

destination → **↓ source**	**1**	**2**	**3**	**4**	**Supply**
1	(2,3,5,6) R=4(5)	(1,4,5,6) R=4(3)	(1,2,4,5) R=3	(1,3,5,7) R=4	8
2	(2,4,5,9) R=5(6)	(6,7,9,10) R=8	(6,8,10,12) R=9	(8,9,11,12) R=10(13)	19
3	(4,5,7,8) R=6	(0,1,3,4) R=2	(2,4,6,8) R=5(14)	(0,1,1,2) R=1(3)	17
demand	11	3	14	16	

For feasible solutions$x_{11} = 5$, $x_{12} = 3$, $x_{21} = 6$, $x_{24} = 13$, $x_{33} = 14$ and $x_{34} = 3$, objective functions are as below: $\tilde{Z}_1 = (46,124,173,229)$ and $\tilde{Z}_2 = (166,224,295,375)$ and $(R(Z_1), R(Z_2)) = (143,265)$.

Table 4

Based on step (4), create rectangular loop (2.4), (2.3), (3.3), (3.4) and (2.4) and obtain reduced table 4.

destination → **↓ source**	**1**	**2**	**3**	**4**	**Demand**
1	(2,3,4,7) R=4(5)	(2,3,5,6) R=4(3)	(1,2,4,5) R=3	(1,3,5,7) R=4	8
2	(3,4,5,8) R=5(6)	(6,7,9,10) R=8	(6,8,10,12) R=9(λ)	(8,9,11,12) R=10$(13 - \lambda)$	19
3	(4,5,7,8) R=6	(0,1,3,4) R=2	(2,4,6,8) R=5$(14 - \lambda)$	(0,1,1,2) R=1$(3 + \lambda)$	17
Demand	11	3	14	16	

For each $\lambda \in \{1,2,\dots,13\}$ value, FFOTP transportation cost is $\tilde{Z}_1 = (46,124,173,229 + 4\lambda)$ and loss cost in SFOTP is:

$$\tilde{Z}_2 = (166 - 4\lambda, 224 - 4\lambda, 295 - 6\lambda, 375 - 6\lambda)$$

where

$$(R(\tilde{Z}_1), R(\tilde{Z}_2)) = (143 + \lambda, 265 - 5\lambda)$$

For maximum λ is 13. Objective function is:

$$\tilde{Z}_1 = (46,124,173,281) \text{ و } \tilde{Z}_2 = (114,172,217,297)$$
$$\text{و } (R(\tilde{Z}_1), R(\tilde{Z}_2)) = (156,200)$$

And feasible solution is:

$$x_{11} = 5, x_{12} = 3, x_{21} = 6, x_{23} = 13, x_{33} = 1,$$
$$x_{34} = 16$$

Table 5
Because feasible solution is not SFOTP ideal solution, we repeat steps (4) and 5.

destination → ↓**source**	**1**	**2**	**3**	**4**	**Supply**
1	(2,3,4,7) R=4$(5-\lambda)$	(2,3,5,6) R=4(3)	(1,2,4,5) R=3(λ)	(1,3,5,7) R=4	8
2	(3,4,5,8) R=5$(6+\lambda)$	(6,7,9,10) R=8	(6,8,10,12) R=9$(13-\lambda)$	(8,9,11,12) R=10	19
3	(4,5,7,8) R=6	(0,1,3,4) R=2	(2,4,6,8) R=5(1)	(0,1,1,2) R=1(16)	17
demand	11	3	14	16	

For each $\lambda \in \{1,\dots,5\}$, FFOTP transportation cost is $\tilde{Z_1} = (46+3\lambda, 124+4\lambda, 173+4\lambda, 281+5\lambda)$ and loss cost in SFOTP is $\tilde{Z_2} = (114-4\lambda, 172-5\lambda, 217-5\lambda, 297-6\lambda)$ where $(R(\tilde{Z_1}), R(\tilde{Z_2})) = (156+4\lambda, 200-5\lambda)$.

For maximum λ which is 5, objective function is:

$$\tilde{Z_1} = (61,144,193,306) \text{ و } \tilde{Z_2} = (94,147,192,267) \text{ و}$$
$$(R(\tilde{Z_1}), R(\tilde{Z_2})) = (176,175)$$

And feasible solution is:

$$x_{12} = 3,\ x_{13} = 5,\ x_{21} = 11,\ x_{23} = 8,\ x_{33} = 1,$$
$$x_{34} = 16$$

Because

Table 6
feasible solution is not optimal solution for SFOTP, we repeat steps (4) and 5.

destination → ↓ **source**	**1**	**2**	**3**	**4**	**Supply**
1	(2,3,4,7) R=4	(2,3,5,6) R=4$(3-\lambda)$	(1,2,4,5) R=3$(5+\lambda)$	(1,3,5,7) R=4	8
2	(3,4,5,8) R=5(11)	(6,7,9,10) R=8(λ)	(6,8,10,12) R=9$(8-\lambda)$	(8,9,11,12) R=10	19
3	(4,5,7,8) R=6	(0,1,3,4) R=2	(2,4,6,8) R=5(1)	(0,1,1,2) R=1(16)	17
demand	11	3	14	16	

For each $\lambda \in \{1,2,3\}$, cost of FFOTP is $\tilde{Z_1} = (61+9\lambda, 144+10\lambda, 193+11\lambda, 306+14\lambda)$ and loss cost of SFOTP is $\tilde{Z_2} = (94-\lambda, 147-2\lambda, 192-2\lambda, 267-3\lambda)$ where $(R(\tilde{Z_1}), R(\tilde{Z_2})) = (176+11\lambda, 175-2\lambda)$.

For maximum λ which is 3, objective function value is $\tilde{Z_1} = (88,174,226,348)$ و $\tilde{Z_2} = (91,141,186,258)$ و $(R(\tilde{Z_1}), R(\tilde{Z_2})) = (209,169)$ and feasible solution is:

$$x_{13} = 8,\ x_{21} = 11,\ x_{22} = 3,\ x_{23} = 5$$
$$,\ x_{33} = 1,\ x_{34} = 16$$

Table 7
Because feasible solution is not SFOTP solution, we repeat steps (4) and 5.

destination →					supply
↓ source	1	2	3	4	
1	(2,3,4,7) R=4	(2,3,5,6) R=4	(1,2,4,5) R=3(8)	(1,3,5,7) R=4	8
2	(3,4,5,8) R=5(11)	(6,7,9,10) R=8$(3-\lambda)$	(6,8,10,12) R=9$(5+\lambda)$	(8,9,11,12) R=10	19
3	(4,5,7,8) R=6	(0,1,3,4) R=2(λ)	(2,4,6,8) R=5$(1-\lambda)$	(0,1,1,2) R=1(16)	17
demand	11	3	14	16	

The only value is $\lambda = 1$; therefore, objective function is:

$$\widetilde{Z_1} = (86,173,225,348), \widetilde{Z_2} = (89,139,184,256)$$
$$(R(\widetilde{Z_1}), R(\widetilde{Z_2})) = (208,167)$$

And feasible solution is:

$$x_{13} = 8, x_{21} = 11, x_{22} = 2,$$
$$x_{23} = 6, x_{32} = 1, x_{34} = 16$$

Because 167 is optimal solution for SFOTP, step (6) ends. Using steps (7) and (8), FP solutions is obtained by SFOTP for FFOTP, is in table (8):

Table 8

Repeat	λ	Solution FP
1	{1,2}	$x_{12} = \lambda, x_{13} = 8-\lambda, x_{21} = 11,$ $x_{22} = 2-\lambda, x_{23} = 6+\lambda, x_{32} = 1, x_{34} = 16$
2	{1}	$x_{12} = 2+\lambda, x_{13} = 6-\lambda, x_{21} = 11,$ $x_{23} = 8, x_{32} = 1-\lambda, x_{33} = \lambda, x_{34} = 16$
3	{1,…,5}	$x_{11} = \lambda, x_{12} = 3, x_{13} = 5-\lambda,$ $x_{21} = 11-\lambda, x_{23} = 8+\lambda, x_{33} = 1, x_{34} = 16$
4	{1,…,13}	$x_{11} = 5, x_{12} = 3, x_{21} = 6, x_{23} = 13-\lambda,$ $x_{24} = \lambda, x_{33} = 1+\lambda, x_{34} = 16-\lambda$

Solutions FP which is obtained from FFOTP to SFOTP and from SFOTP to FFOTP and satisfaction level functions from efficient solution are presented in table (9).

Table 9
Above table helps decision-maker to determine preferred solution among efficient solutions.

	Objective function FP			Satisfaction	
	$\widetilde{Z_1}$	$\widetilde{Z_2}$	$(R(\widetilde{Z_1}), R(\widetilde{Z_2}))$	FFOTP	SFOTP
1	(46,124,173,229)	(166,224,295,375)	(143,265)	100	41.30
2	(46,124,173,233)	(162,220,289,369)	(144,260)	99.30	44.31
3	(46,124,173,237)	(158,216,283,363)	(145,255)	98.60	47.31
4	(46,124,173,241)	(154,212,277,357)	(146,250)	97.90	50.30
5	(46,124,173,245)	(150,208,271,351)	(147,245)	97.20	53.29
6	(46,124,173,249)	(146,204,265,345)	(148,240)	96.50	56.29
7	(46,124,173,253)	(142,200,259,339)	(149,235)	95.80	59.28
8	(46,124,173,257)	(138,196,253,333)	(150,230)	95.10	62.28
9	(46,124,173,261)	(134,192,247,327)	(151,225)	94.41	65.27
10	(46,124,173,265)	(130,188,241,321)	(152,220)	93.71	68.26

11	(46,124,173,269)	(126,184,235,315)	(153,215)	93.01	71.26
12	(46,124,173,273)	(122,180,229,309)	(154,210)	92.31	74.25
13	(46,124,173,277)	(118,176,223,303)	(155,205)	91.61	77.25
14	(46,124,173, 281)	(114,172,217,297)	(156,200)	90.91	80.24
15	(49,128,177,234)	(110,167,212,291)	(160,195)	88.11	83.23
16	(52,132,181,239)	(106,162,207,285)	(164,190)	85.31	86.23
17	(55,136,185,244)	(102,157,202,279)	(168,185)	82.52	89.22
18	(58,140,189,249)	(98,152,197,273)	(172,180)	79.72	92.22
19	(61,144,193,254)	(94,147,192,267)	(176,175)	76.92	95.21
20	(77,163,214,334)	(90,141,186,259)	(186,171)	69.93	97.60
21	(70,154,204,320)	(93,145,190,264)	(187,173)	inefficient	solution
22	(68,153,203,320)	(91,143,188,262)	(197,169)	62.24	98.80
23	(79,164,215,334)	(79,164,215,334)	(198,171)	inefficient	solution
24	(88,174,226,348)	(91,141,186,258)	(209,169)	inefficient	solution
25	(86,173,225,348)	(89,139,184,256)	(208,168)	54.55	100

7. Conclusion

In this article, proposed method produces efficient solutions of fuzzy two-objective transportation problem with ranking function which obtained efficient solutions using efficient solutions. This method is different with other methods that have been used for solving fuzzy multi-objective transportation problems that decision maker can determine the preferred solution from efficient solution using it.

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