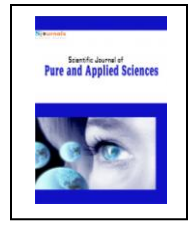
Sjournals
Scientific Journals
Scientific Journal of Pure and Applied Sciences (2013) 2(4) 190-195
ISSN 2322-2956

Contents lists available at Sjournals
Scientific Journal of
Pure and Applied Sciences
Journal homepage: www.Sjournals.com



Original article

Tawanda's allocation method for the 0-1 knapsack problem

T. Tawanda

Department of Applied Mathematics-Operations Research and Statistics, National University of Science and Technology, PO Box AC 939, Ascot, Bulawayo, Zimbabwe.

[*]Corresponding author; Department of Applied Mathematics-Operations Research and Statistics, National University of Science and Technology, PO Box AC 939, Ascot, Bulawayo, Zimbabwe.

ARTICLE INFO

Article history:
Received 27 March 2013
Accepted 25 April 2013
Available online 28 April 2013

Keywords:
Knapsack
Allocation
Optimal

ABSTRACT

In this paper, a new allocation method to solve the knapsack problems is developed and demonstrated. The method makes use of all possible item combinations to produce the optimal solution. The allocation method is divided into two sub - allocations procedures namely, the initial allocation procedure and the objective allocation procedure. Existence of combinations is determined by the initial allocation whereas the optimality of allocation is determined by the objective allocation. The method is capable of computing all possible solutions to the problem.

© 2013 Sjournals. All rights reserved.

1. Introduction

The basic idea of the knapsack problem is that there are N different types of items that can be put into a knapsack. Each item has a certain weight associated with it as well as a value. The problem is to determine how many units of each item to place in the knapsack (Anderson, et al 2000). According to (Ross et al, 1976)the objective is to maximize the total value of the content carried, given various weight, volume and cost constraint. The knapsack problem has many applications in the real word, such as in naval applications, cargo loading, the optimal investment plan etc. There are many types of the knapsack problems. The 0 – 1 knapsack considers a number of items, each with a known weight and value, with a goal of maximizing the value obtained by selecting a subset of items whose collective weight does not exceed the limit (Martello and Toth, 1990). Burkard and Pferschy (1994) considered the inverse knapsack problem. The branch and bound approach were applied to solve the 0 – 1 knapsack problems, where we consider the 2^n possible vectors X, the profit is calculated whilst keeping track of the highest found and the corresponding vector (Bulfin et al; 1979). Morin and Marsten (1976) as quoted by (Agrali and Geunes, 2009) discussed a dynamic programming approach for discrete, non-linear and separable knapsack problems, where no convexity or concavity assumptions are made on the objective function which they assume to be non-decreasing in the decision variable. The nested knapsack problem was considered by Dudzinski and Walukiewicz (1984). When the items should be chosen from disjoint classes, and if knapsack problems are to be filled simultaneously, we get the multiple knapsack problems, this type was also considered by (Dyer et al, 1984). Nauss (1978)proposed to transform non – linear knapsack problems to multi-choice problems. Fayard and plateau (1994) considered the collapsing knapsack problems. Bin packing problem is also classified as the knapsack problem, where the set of all the objects needs to be partitioned into subsets, bins such that the total weight in each bin is less than the capacity, and the number of bins needed is minimized.

2. The allocation method

2.1. The initial allocation procedure

For a combinations to exist we allocate an allocation called the initial allocation procedure, thus allocating a single item for every item type defined in a combination. After initial allocation, if the combination does not satisfy the knapsack conditions, thus the total weight of the combination after initial allocation is greater than the capacity of the knapsack, then the combination is said to exist but doesn't hold. A combinations holds if and only if the total weight after the initial allocation is less than or equal to the knapsack capacity. All combinations that hold are called the feasible combinations and the optimal combination is among the feasible combinations.

2.2. Objective allocations procedure

After an initial allocation, we allocate another allocation called the objective allocation procedure. This allocation procedure is done in such a way that the objective of the knapsack problem is optimized thus, profit maximization or cost minimization hence the name. We allocate it after playing around with weight of sub-combinations of a specific combination obtained by 2^k combination method so as to find the sub-combinations of a specific combination that gives us the optimal solution.

2.3. Parameters

x_i Items to be transported, $i = 1, 2, 3, \ . \ . \ . \ , n$

w_j Respective unit weight of itemx_i,$j = 1, 2, 3, \ . \ . \ . \ , n$ and $j = i$

W The knapsack capacity

N_s The number of units available of itemx_i,$s = 1, 2, 3, \ . \ . \ . \ , n$, where$s = i$

r_i The calculated numeric value of x_i

a_i The weight of items added to w_j of item typex_i, where$(i, j) = 1, 2, 3, \ . \ . \ . \ , n$

∏ The difference between knapsack capacity and the initial allocation weight

2.4. The general knapsack problem formulation

The appropriate mathematical formulation is given by

$$\text{Maximize } \sum_{i=1}^{n} r_i x_i \quad (1)$$

Subject to $\sum_{i=1}^{n} w_i x_i \leq$ W

The general weight limitation can be expressed by setting the sum of weights of all items that are considered in a combination as follows,

$w_1x_1 + w_2x_2 + w_3x_3 + \ldots. + w_nx_n \leq$ W (2)

We can apply 2^k combinations to obtain all possible item combinations that can satisfy the general weight constraint above. The above inequality (2) shows the unit allocation after initial allocation.

We can apply the objective allocation to (2), thus the resulting inequality will be expressed as follows.

$(w_1x_1 + w_2x_2 + w_3x_3 + \ldots + w_nx_n) + (a_1x_1 + a_2x_2 + a_3x_3 + \ldots + a_nx_n) \leq$ W (3)

The above expression can be written as follows

$(w_1 + a_1)x_1 + (w_2 + a_2)x_2 + (w_3 + a_3)x_3 + \ldots + (w_n + a_n)\,x_n \leq$ W (4)

From (4), we can calculate the total number of units contributed by each item typex_i in a combination as follows,

$$\frac{\text{Initial allocation} + \text{Objective allocation}}{\text{Unit weight}} \leq \text{Number available}$$

Mathematically the above expression can be expressed as below,

$$\left[\frac{w_i + a_i}{w_i}\right] \leq N_i(\mathbf{5})$$

The coefficients of (4), has to satisfy the above inequality.

2.5. Combination return calculations

Let us consider the general combination**(4)**given by

$(w_1 + a_1)x_1 + (w_2 + a_2)x_2 + (w_3 + a_3)x_3 + \ldots + (w_n + a_n)\,x_n \leq$ W

We can compute the return of this combination as illustrated below.

$$\text{Return} = \left[\frac{w_1 + a_1}{w_1}\right]r_1 + \left[\frac{w_2 + a_2}{w_2}\right]r_2 + \left[\frac{w_3 + a_3}{w_3}\right]r_3 + \ldots\ldots + \left[\frac{w_n + a_n}{w_n}\right]r_n(\mathbf{6})$$

3. Results

Lemma 1

The combination does not hold if,

W - $(w_1x_1 + w_2x_2 + w_3x_3 + \ldots + w_nx_n)$ = ∏, ∏ < 0

Proof

Since such combination is greater than the knapsack capacity W after allocating a unit ofx_i, thus for such a combination to hold we have to remove at least one type of items involved in the defined combination. By so doing the resulting combination is an element of $\mathbf{2^k}$combinations that already exists.

Lemma 2

The objective allocation does not exist if,

W - ($w_1x_1 + w_2x_2 + w_3x_3 + \ . \ . \ . + w_nx_n$) = ∏, ∏ = 0

Proof

If the initial allocation is equal to the knapsack capacity W, this means that the combination is optimal and there is no space in the knapsack to add other units, thus the objective allocation does not exist.

Lemma 3

The objective allocation exist if,

W - ($w_1x_1 + w_2x_2 + w_3x_3 + \ . \ . \ . + w_nx_n$) = ∏, ∏ > 0, suchthat at least one w_j ≤ ∏

Proof

Given that ∏ > 0, and at least one w_j ≤ ∏, then ∃ objective allocation procedure combination of the form, ($a_1x_1 + a_2x_2 + a_3x_3 + \ . \ . \ . + a_nx_n$), where,

$a_1x_1 + a_2x_2 + a_3x_3 + \ . \ . \ . + a_nx_n$ ≤∏, if and only if∃ at least one w_j : w_j ≤ ∏

3.1. Example

Table below summarizes important information about three items that has to be transported by a 'knapsack'. The total capacity of the knapsack is 10 tons. Weight of items is given in tons, and profit is in dollars and the number available per each item.

Table 1
The table above shows the knapsack problem.

Item	Weight per unit (tons)	Profit per unit (dollars)	Number of units available
x_1	1	3	6
x_2	4	9	1
x_3	3	8	2
x_4	2	5	2

3.2. Solution

The objective is to find the total weight that can be transported at a maximum profit without breaking the knapsack.

The maximum return from our solution is $ 28, and in this case we have two combinations that give us an optimal solution. The combinations are:

1. Combination x_1x_3, when we transport 4 units of x_1 and 2 units of x_3. The total profit is $ 28.
2. Combination x_1x_4, when we transport 6 units of x_1 and 2 units of x_4. The total profit is $ 28.

3.3. Solution method guide lines

1. All allocations are in terms of weight, when allocating the initial allocation we allocate respective weight per unit to all items that makes up a combination. When the total weight of the initial allocation is less than the knapsack capacity we may or may not allocate depends on the space left ∏, thus the difference between the initial allocation and the knapsack capacity. We can only allocate

if there exist a weight per unit among the items that makes up a combination w_j such that $w_j \leq \prod$. When allocating the objective allocation items in a fashion that gives us the maximum or minimum return depending on our objective allocation. At the same time we make sure that the initial allocation plus the objective allocation does not exceed the knapsack size. On the other hand the total number of units allocated to the initial and the objective allocations for every x_i does not exceed the number of units available for the respective x_is in every combination.

Table 2

shows all possible combinations and item allocations in example 3.1, the optimal combination is the one with the highest total return.

Item combination	Initial + objective allocation (tons)	Total weight per combination (tons)	Total profit per combination ($)
x_1	$(x_1) + (5x_1)$	$6x_1$	18
x_2	$(4x_2) + (0x_I)$	$4x_2$	9
x_3	$(3x_3) + (3x_3)$	$6x_3$	16
x_4	$(2x_4) + (2x_4)$	$4x_4$	10
x_1x_2	$(x_1 + 4x_2) + (5x_1)$	$6x_1 + 4x_2$	27
x_1x_3	$(x_1 + 3x_3) + (3x_1 + 3x_3)$	$4x_1 + 6x_3$	28
x_1x_4	$(x_1 + 2x_4) + (5x_1 + 2x_4)$	$6x_1 + 4x_4$	28
x_2x_3	$(4x_2 + 3x_3) + (3x_3)$	$4x_2 + 6x_3$	25
x_2x_4	$(4x_2 + 2x_4) + (2x_4)$	$4x_2 + 4x_4$	19
x_3x_4	$(3x_3 + 2x_4) + (3x_3 + 2x_4)$	$6x_3 + 4x_4$	26
$x_1x_2x_3$	$(x_1 + 4x_2 + 3x_3) + (2x_1)$	$3x_1 + 4x_2 + 3x_3$	26
$x_1x_2x_4$	$(x_1 + 4x_2 + 2x_4) + (x_1 + 2x_4)$	$2x_1 + 4x_2 + 4x_4$	25
$x_1x_3x_4$	$(x_1 + 3x_3 + 2x_4) + (x_1 + 3x_3)$	$2x_1 + 6x_3 + 2x_4$	26
$x_2x_3x_4$	$(4x_2 + 3x_3 + 2x_4) + (0x_i)$	$4x_2 + 3x_3 + 2x_4$	22
$x_1x_2x_3x_4$	$(x_2 + 4x_2 + 3x_3 + 2x_4) + (0x_i)$	$x_2 + 4x_2 + 3x_3 + 2x_4$	25

4. Discussion

The allocation method can compute the optimal solutions to all possible combinations in the knapsack problem, this gives flexibility in terms of decision making in the sense that it is not always the case that the optimal solution is the bestdecision to consider. The method is exponential in nature it expands outwards as number of items involved in the problem increases, however higher order combinations tends to approach or exceed the knapsack capacity after initial allocation hence the chances of higher order combinations to hold after initial allocation tends to zero.Comparing the allocation method with the dynamic approach to the knapsack problem, it is clear that the allocation method performs better in knapsack problems with a few items since it uses the non-iterative approach to the problems.

5. Conclusion

For further research the author suggests the implementation of a computer program that will solve the knapsack problems using the allocation method methodology thus, initial and objective allocation procedures.

Acknowledgment

The author would like to thank Dr S.Mushayabasa and Dr P. Bhunu at the University of Zimbabwe and Mr. C N. Mupondo at National University of Science and Technology, Zimbabwe for their helpful comments and suggestions during the development of this paper.

References

Agralı, S., Geunes, J., 2009. Solving knapsack problems with S-curve return functions, Europ. J. Opera. Res., 193, 605–615.

Anderson, D.R., Sweeney, D.J., Williams, T.A., 2000. Quantitative methods for business. south-west college publishing, pp. 829-834.

Bulfin, R.L., Parker, R.G., Shetty, C.M., 1979. "Computational results with a branch and bound algorithm for general knapsack problem". Naval Res. Log. Quart., 26, 41 – 46.

Burkard, R.E., Pferschy, U., 1994. "The Inverse parametric Knapsack Problem", Technishe Universitat Graz, Australia, Report 94/288.

Dudzinski, K., Walukiewicz, S., 1984. "Exact Method for the Knapsack Problem and its Generalizations", Europ. J. Opera. Res., 28, 3 – 21.

Dyer, M.E., 1984. "An O (n) lgorithm for the multiple- choice knapsack linear program". Math. Program., 29, 57 – 63.

Fayard, D., Plateau, G., 1994. "An exact algorithm for the 0 – 1 collapsing knapsack problem". Dis. Appl. Math., 49, 175-187.

Harvey, M., Warger, 1969. Principles of operations research, prentice hall Inc., pp. 349 – 353.

Martello, S., Toth, P., 1990. " Knapsack Problems – Algorithms and Computer Implementations", Wiley, New York, NY, 1990.

Morin, T.L., Marsten, R.E., 1976. "An algorithm for nonlinear knapsack problems. Manag. Sci., 22 (10), 1147–1158.

Nauss, R.M., 1987. ''The 0 – 1 Knapsack problem with multiple choice constraint''. Europ. J. Opera. Res., 2, 125 – 131.

Ross, H., Johnson, Paul, R., Winn, 1976. Quantitative methods for management. HoughtonMifflin company, pp. 337 – 340.